INPUT IMPEDANCE OF HIGH-FREQUENCY PARALLEL WIRE TRANSMISSION LINES IMMERSED IN AN ABSORBING MEDIUM *

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ABSTRACT. The input impedance of high frequency parallel wire transmission lines, quarter wavelength long, has been mathematically calculated when the lines are immersed in an absorbing medium. The calculated values have been verified by actually determining the impedance of such lines immersed in dry soil. The impedance has been determined for lines short-circuited at their far ends and also when these ends are open. The method of performing the experiments has been described. The measurements have been made within the frequencies of 31 megacycles / sec to 114 megacycles / sec. The results obtained have been compared with those when there is no absorbing medium between the two parallel wires. It has been observed that within the range of the frequencies employed for observations, the input impedance of such system of quarter-wave lines remains nearly equal to the surge impedance of the lines at higher frequencies, when the attenuation is fairly high, for both open as well as close terminated lines. It tends to increase as the frequency is lowered and the attenuation constant is below 3×10^{-2}, in the case of the lines with short-circuited termination. For the open-ended lines however, the impedance decreases as the frequency is reduced.

INTRODUCTION

Parallel-wire high frequency transmission lines have gained a very wide application within the last decade in connection with communications by means of ultra-short radio waves. In most of the cases, such lines have been used for the purpose of transferring energy from one part of the apparatus to the other or as an impedance matching transformer. Recently, however, such transmission lines have been employed by the present author and his collaborators (1,2,3) for the determination of various electrical constants of an absorbing medium like soil and ionized gas at ultra-high frequencies. In whatever form the transmission lines may be used, the knowledge of input impedance of such a system is essential for proper functioning of the lines. The impedance of such lines for different lengths, when placed in air, has been worked out by various authors (4,5) but attention has not been directed to its effective value when the lines are immersed in an absorbing medium. In the present paper the input impedance of parallel-wire high-frequency transmission lines, quarter-wavelength long, with

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open as well as short-circuited terminations, has been worked out mathematically and the values thus obtained have been verified experimentally. The results have been compared with those when there is no absorbing medium between the wires The medium used was well-sieved dry soil as employed in the previous experiments (1) for the determination of electrical constants of soil. The observations were made with the frequencies between 31 megacycles per second to 114 megacycles per second.

THEORY

The input impedance of a system of parallel wire transmission lines is given by,

$$Z_i = \left[\frac{Z_a \cosh l\,(A+jB) + Z_o \sinh l\,(A+jB)}{Z_o \cosh l\,(A+jB) + Z_a \sinh l\,(A+jB)}\right] \tag{1}$$

where,
A = Attenuation constant,
B = Wavelength constant,
Z_o = Surge impedance of the lines,
Z_a = Impedance at the far end of the lines,
l = Geometrical length of the lines.

Let us now consider the condition when there is enough absorption due to the presence of soil or any such medium between the two wires, so that the attenuation may not be neglected. By expanding the terms in equation (1) and after further simplification, we get,

$$Z_i = Z_o \left[\frac{Z_a\{\cosh lA \cos lB + j \sinh lA \sin lB\} + Z_o\{\sinh lA \cos lB + j \cosh lA \sin lB\}}{Z_o\{\cosh lA \cos lB + j \sinh lA \sin lB\} + Z_a\{\sinh lA \cos lB + j \cosh lA \sin lB\}}\right] \quad \ldots \tag{2}$$

For the sake of convenience, lines quarter-wavelength long have been used for the measurements and therefore, the impedances for such lengths only have beed discussed below.

For lines quarter wavelength long, we have $lB = \pi/2$ and substituting this value of lB in equation (2), the input impedance of the lines will be given by,

$$Z_i = Z_o \frac{Z_a \sinh lA + Z_o \cosh lA}{Z_o \sinh lA + Z_a \cosh lA} \tag{3}$$

The input impedance of the lines for two terminal conditions have been considered. First, when the line is short-circuited at its far end and the second when it is terminated with open ends.

Case (1). Quarter-wave long transmission lines with short-circuited termination.

When the line is short-circuited at its far end, the terminal impedance $Z_a = 0$. and substituting this value of Z_a in equation (3), the input impedance of the line will be given by,

$$Z_i = Z_o \coth (A\lambda/4) \qquad \ldots \quad (4)$$

Some interesting points may be noted in equation (4). When attenuation is large, which is true for most of the practical cases at higher frequencies, we have the input impedance nearly equal to the surge impedance of the line. But when there is no absorbing medium between the parallel wires, we have the input impedance, $Z_i = \infty$. obtained from the well-known relation $Z_i = Z_o^2/Z_a$. Thus we may conclude that whenever there is an absorbing medium between the line wires, the input impedance of the line, quarter-wavelength long and short-circuited at the far end, ceases to be very high and for most of the practical purposes it is equal to the surge impedance of the line. This has been experimentally verified as shown in the later section.

Case (2). Quarter-wave long transmission lines with open-circuited termination.

In this case the terminal impedance of the lines, $Z_a = \infty$. Substituting this value of Z_a in equation (3), the input impedance of the line will be given by,

$$Z_i = Z_o \tanh (A\lambda/4) \qquad \ldots \quad (5)$$

It will be observed from equation (5) that for greater attenuation, the impedance of such line becomes equal to the surge impedance of the line. When there is no absorbing medium between the line wires, the input impedance of such line vanishes. The values of the input impedance calculated from equation (5) were verified experimentally as shown later.

It will be further noted from equation (3) that when the quarter-wave line immersed in soil is terminated at its far end with a resistance equivalent to the surge impedance, Z_o, of the line, the input impedance remains equal to the surge impedance under all conditions. This is similar to the case of the line without being immersed in an absorbing medium.

EXPERIMENTAL ARRANGEMENTS AND OBSERVATIONS

The soil after being properly sieved was kept in an open, rectangular, long wooden box of dimensions, 250 cm. × 11 cm. × 10 cm. with two smaller sides made of ebonite. Two bare copper wires, no. 14 s.w.g., were fixed on one of the small ebonite sides with binding screws and the wires were made tight and to run parallel along the length of the box. They were terminated on the other small ebonite side and were attached to two adjustable hooks for proper tension. The wires were placed 5 cm. apart from each other. When the measurements were taken with lines closed at its far end, the line wires were short-circuited with a metal bridge M (Fig. 1) at that end. The line wires LL were completely covered with soil pressed by its own weight. The wires could be connected by means of a double-pole double-throw switch S to a small variable micro-condenser C, an inductance loop loosely coupled with a valve-generator G and a radio-frequency thermo-galvanometer T. The valve-generator could emit out waves of lengths 2 metres to 10 metres as required by the experiment.

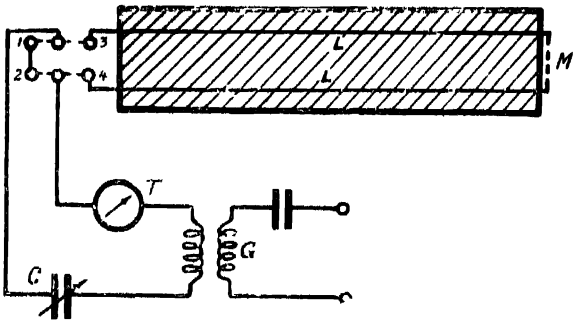

FIGURE 1

The switch S is thrown on the terminals 1, 2 (Fig. 1) and the circuit is tuned for resonance at the particular frequency generated by the valve oscillator with the help of the variable condenser C. The switch is then thrown on 3, 4 and again the resonance capacity is observed in the micro-condenser. The resonance conditions are observed by the maximum response in the thermo-galvanometer. If C_1 and C_2 be the two values of the capacities at resonance, the line impedance is computed from $\omega C_1 \quad \omega C_2$, where ω is the angular frequency of the waves.

As the values of the capacities C_1 and C_2 were both small, being only some fraction of the micro-condenser itself, the exact magnitudes of these were determined by a separate Lecher wire system with the same frequencies at which the observations were taken for finding out the resonance capacities. The micro-condenser was connected to one end of a pair of Lecher wires and positions of current antinodes were observed by a thermo-galvanometer. The capacity was calculated as shown below, from the difference between the length measured from the end containing the condenser to the first antinode and the length between two consecutive antinodes. If the difference in the length is denoted by δl, the unknown capacity C_x can be calculated from the relation,

$$C_x = \frac{lC}{\pi} \tan \frac{\pi \delta l}{l}$$

where l = distance between two consecutive antinodes,
C = capacitance of the Lecher wires.

A calibration graph was drawn from different values of the micro-condenser C from which any value of C_x could be determined knowing the corresponding value of δl. The attenuation constant of the soil was measured at different frequencies by the method applied by us (1) in the previous experiments on electrical constants of soil.

Table I below gives the attenuation constants determined at the wavelengths used in the experimental investigation.

Surge impedance of the line = 470 ohms

TABLE I

Frequency in megacycles/sec.	Attenuation constant.
113·64	$9{\cdot}68 \times 10^{-2}$
93·75	6·41 ,,
81·08	5·38 ,,
69·42	4·21 ,,
63·16	3·15 ,,
57·91	2·12 ,,
49·83	1·01 ,,
40·00	$6{\cdot}20 \times 10^{-3}$
32·61	4·10 ,,

Table II below gives the input impedance of the transmission lines immersed in soil, quarter-wavelength long and short circuited at the far end. Table III gives the same for transmission lines terminated with open ends. Second column of these tables shows the calculated values and the third column indicates the observed ones.

TABLE II

Frequency in megacycles/sec.	Input impedance in ohms calculated from $Z_s = Z_0 \coth(A\lambda/4)$	Input impedance in ohms observed experimentally.
113·64	470·0	461·4
93·75	470·0	466·7
81·08	470·0	455·4
69·42	470·1	452·8
63·16	470·7	479·6
57·91	473·3	479·6
49·83	518·2	532·2
40·00	572·5	596·8
32·61	639·5	632·0

3

TABLE III.

Frequency in megacycles/sec.	Input impedance in ohms calculated from $Z_i = Z_0 \tanh. (A\lambda/4)$.	Input impedance in ohms observed experimentally.
113·64	470·0	463·6
93·75	470·0	474·3
81·08	470·0	462·8
69·42	469·9	459·4
63·16	469·3	455·8
57 91	466·1	455·0
49·83	426·3	410·3
40·00	385·9	404·6
32·61	345·4	312·3

Comparing the values of attenuation constant from Table I with those of the impedance of the lines given in Tables II and III for the same frequencies, it will be observed that for higher frequencies, when the attenuation is considerable, the impedance of the quarter-wave long transmission lines with open as well as closed terminations, remains equal to the surge impedance of the lines. As the frequency is lowered till the attenuation constant goes below 3×10^{-2}, the impedance of the lines with closed termination gradually increases and that of the line with open termination decreases.

The variations of input impedance of quarter-wave long transmission lines for short-circuited and open-circuited terminations, when they are immersed in soil, have been shown graphically in Figs. 2 and 3 respectively. The continuous

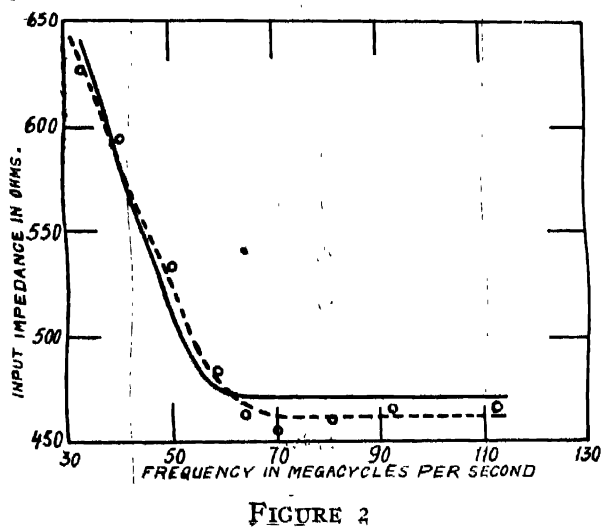

FIGURE 2

lines have been drawn from the calculated values and dotted ones from the experimentally observed results.

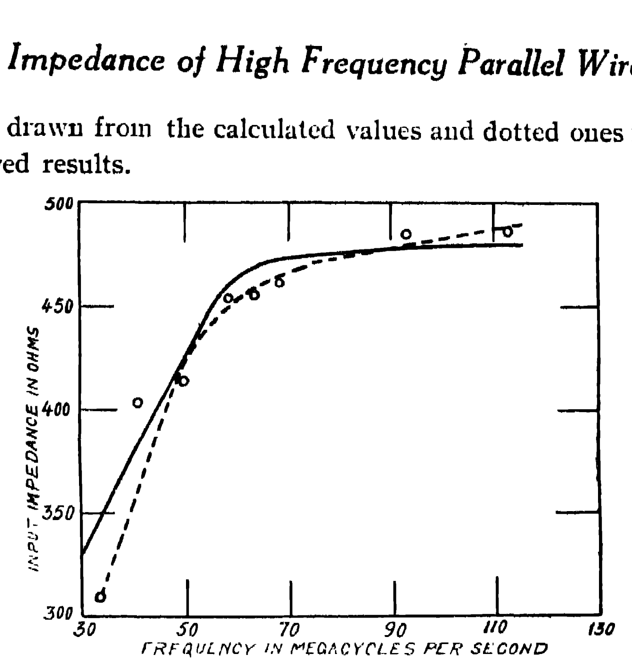

FIGURE 3

SUMMARY AND CONCLUSIONS

The input impedance of parallel-wire high-frequency transmission lines quarter-wavelength long has been mathematically calculated and verified by experiments. Lines have been used with their far ends open and also when they are short-circuited. Results obtained have been compared with those when the lines are not immersed in an absorbing medium. The frequencies used for the purpose of measurement of the impedance were between 31 megacycles/sec. to 114 megacycles/sec. It has been observed that within this range of frequencies the input impedance of quarter-wave long transmission lines with open as well as closed terminations remains nearly the same as the surge impedance of the lines at higher frequencies, when the attenuation is considerable. At lower frequencies, however, when the attenuation constant decreases below 3×10^{-2}, the impedance of the lines with far ends short-circuited, increases as the frequency is lowered. For lines with open termination, the impedance decreases as the frequency is reduced.

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